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1. **Summary Statistics of Sample Data in one table**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Derivation** | **Sample1** | **Sample 2** | **Sample 3** | **Sample 4** |
| **Mean** | 11.9587 | 12.0287 | 11.8890 | 12.0813 |
| **Median** | 11.9550 | 12.0250 | 11.9200 | 12.0800 |
| **Standard Deviation** | 0.2204 | 0.2204 | 0.2072 | 0.2062 |

1. **Hypothesis Testing for all the samples**

The hypothesis test suggests by quality associates follow.

H 0: μ = 12

H a: μ ≠ 12

Level of confidence alpha = 0.05

Population Mean = 12

Population Standard Deviation = 0.21

Sample Size of all samples = 30 each

To cross validate the sample data we ran qq plot and noticed if the sample data follows linear trajectory to confirm if the data is normally distributed or not. Histogram also showed the similar observation, however we are just plotting qq.

A group of graphs showing different types of statistical data

Description automatically generated with medium confidence

Based on above data points and observation we can calculate the Test statistics and its area outside tail region to determine if a null hypothesis can be a reject or do not reject

Test Statistic = (sample mean – population mean)/ (population standard Deviation/ sqrt (sample size))

With the above value we can calculate the area outside the tail regions called a p-value. If the p-value is greater than 0.05 (level of confidence) we can say that we do not reject the null hypothesis and if its <. 0.05 we can say that we can reject the null hypothesis.

Below if the tabular representation of the information.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Sample** | **Sample Mean** | **Z-test Statistic** | **p-Value (Area outside the tails)** | **If greater than alpha 0.05** | **Hypothesis Conclusion** |
| **Sample1** | 11.9587 | -1.0781 | 0.2810 | Yes | Do not reject the null hypothesis (H0 is supported). |
| **Sample2** | 12.0287 | 0.7477 | 0.4546 | Yes | Do not reject the null hypothesis (H0 is supported). |
| **Sample3** | 11.8890 | -2.8951 | 0.0038 | No | Reject the null hypothesis (Ha is supported) |
| **Sample4** | 12.0813 | 2.1213 | 0.0339 | No | Reject the null hypothesis (Ha is supported) |

As evident from the results it is **recommended to take corrective actions on sample 3 and sample 4.** Another analysis on sample 3 data reveals that it has outlier data points as well, however we could not find any such outlier data in any of the other samples.

Note- refer the R-code to check the boxplot of sample 3 to justify our claim of outlier data point.

1. **Does a Population standard deviation of 0.21 seem reasonable?**

Surface level Approach

Given the 4 samples produced a standard deviation in the range of **0.2061 to 0.2203** I think we can say this is a good estimation. Given the consistency of the results among the 4 samples of the same size it seems like this is a pretty good estimate.

Systematic Approach

To achieve it we can validate the above assumption we can define the test hypothesis as

**H0 -** null hypothesis as population standard deviation of 0.21

**Ha** - as population standard deviation not equal to 0.21.

We define the level of significance as 0.05 to assume 95% of confidence in the sample data.

Now calculating the test statistics using chi square distribution

Chi-square X2 = ((n-1). s\*s) /sigma square

n is the sample size

S is sample standard deviation

Sigma is assumed population distribution.

We can calculate the p-value for each and if it’s less than the value of the 1-level of significance we can reject the null hypothesis.

Here is my output →

P-value = 2\*(1- probability at chi-square at n-1 degree of freedom)

Do not reject the null hypothesis: Population standard deviation is reasonable at 0.21.

Chi-Square Statistic: 31.9308

**P-Value: 0.6458**

Do not reject the null hypothesis: Population standard deviation is reasonable at 0.21.

Chi-Square Statistic: 31.9308

**P-Value: 0.6458**

Do not reject the null hypothesis: Population standard deviation is reasonable at 0.21.

Chi-Square Statistic: 28.2238

**P-Value: 1.0119**

Do not reject the null hypothesis: Population standard deviation is reasonable at 0.21.

Chi-Square Statistic: 27.9353

**P-Value: 1.0427**

As the outcome suggests, **it's reasonable to assume the population standard deviation of 0.21**.

1. **Lower Limit and Upper Limit for quality control purposes**

Limit can be calculated by below formula –

Population Mean +- standard error \* z value of alpha/2

Where Standard error is population standard deviation / sqrt of sample size.

With above rule the calculated lower limit and upper limit of each sample is detailed below. As we can notice the sample mean of Sample 3 and 4 falls out of bounds of lower control limit and upper control limit.

Upper Control Limit **(UCL): 12.0751**

Lower Control Limit **(LCL): 11.9248**

1. **Implications of Changing the level of Significance**

Theoretically, it is observed that with a lower alpha (level of significance) we are less likely to reject the null hypothesis leading to lower Type 1 error and by increasing the level of significance we are more likely to reject the null hypothesis leading to higher type 1 type error.

Below is the analysis of hypothesis testing for different confidence levels on the dataset provided to practically see it in action-

|  |  |
| --- | --- |
| **Significance Level (alpha): 0.01** | **Sample: Sample\_1**    Test Statistic: -1.0274    P-Value: 0.3127    Null Hypothesis Not Rejected: **No significant evidence of a difference.**  **Sample: Sample\_2**    Test Statistic: 0.7125    P-Value: 0.4818    Null Hypothesis Not Rejected: **No significant evidence of a difference.**  **Sample: Sample\_3**    Test Statistic: -2.9346    P-Value: 0.0065    Null Hypothesis Rejected: **There is evidence of a difference.**  Sample: Sample\_4    Test Statistic: 2.1614    P-Value: 0.0391    Null Hypothesis Not Rejected: **No significant evidence of a difference.** |
| **Significance Level (alpha): 0.05** | **Sample: Sample\_1**    Test Statistic: -1.0274    P-Value: 0.3127    Null Hypothesis Not Rejected: **No significant evidence of a difference.**  **Sample: Sample\_2**    Test Statistic: 0.7125    P-Value: 0.4818    Null Hypothesis **Not Rejected: No significant evidence of a difference.**  Sample: Sample\_3    Test Statistic: -2.9346    P-Value: 0.0065    Null Hypothesis Rejected: **There is evidence of a difference.**  Sample: Sample\_4    Test Statistic: 2.1614    P-Value: 0.0391    Null Hypothesis Rejected: **There is evidence of a difference.** |
| **Significance Level (alpha): 0.1** | **Sample: Sample\_1**    Test Statistic: -1.0274    P-Value: 0.3127    Null Hypothesis Not Rejected: **No significant evidence of a difference.**  **Sample: Sample\_2**    Test Statistic: 0.7125    P-Value: 0.4818    Null Hypothesis Not Rejected: **No significant evidence of a difference.**  **Sample: Sample\_3**    Test Statistic: -2.9346    P-Value: 0.0065    Null Hypothesis Rejected: **There is evidence of a difference.**  **Sample: Sample\_4**    Test Statistic: 2.1614    P-Value: 0.0391    Null Hypothesis Rejected: **There is evidence of a difference.** |
| **Significance Level (alpha): 0.15** | **Sample: Sample\_1**    Test Statistic: -1.0274    P-Value: 0.3127    Null Hypothesis Not Rejected: **No significant evidence of a difference.**  **Sample: Sample\_2**    Test Statistic: 0.7125    P-Value: 0.4818    Null Hypothesis Not Rejected: **No significant evidence of a difference.**  **Sample: Sample\_3**    Test Statistic: -2.9346    P-Value: 0.0065    Null Hypothesis Rejected: **There is evidence of a difference.**  **Sample: Sample\_4**    Test Statistic: 2.1614    P-Value: 0.0391    Null Hypothesis Rejected: **There is evidence of a difference.** |

Also, we understand that when the population mean moves closer to the null hypothesis value, we have higher chances of making Type 2 errors. Since, it’s not in the scope to change the mean value we will just stick with Type 1 error analysis.

**6. Graph with Sample Mean Trend, population Mean, upper Limit and Lower Limit for all samples**.

A graph with lines and dots

Description automatically generated